Pure Mathematics P1

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

Pure Mathematics P2

Arithmetic series

$u_n = a + (n - 1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Binomial series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$, where $h = \frac{b - a}{n}$

Pure Mathematics P3

Candidates sitting Pure Mathematics P3 may also require those formulae listed under Pure Mathematics P1 and P2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

$$\sin A + \sin B \equiv 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B \equiv 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B \equiv 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B \equiv -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration (+ constant)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$

Pure Mathematics P4

Candidates sitting Pure Mathematics P4 may also require those formulae listed under Pure Mathematics P1, P2 and P3.

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \qquad (\lvert x\rvert < 1,\ n \in \mathbb{R})$$

Integration (+ constant)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert, \quad \ln\left\lvert\tan\left(\frac{1}{2}x\right)\right\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert, \quad \ln\left\lvert\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Further Pure Mathematics FP1

Candidates sitting Further Pure Mathematics FP1 may also require those formulae listed under Pure Mathematics P1 and P2.

Summations

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0 : x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	Rectangular Hyperbola
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \frac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Further Pure Mathematics FP3

Candidates sitting Further Pure Mathematics FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Pure Mathematics P1, P2, P3 and P4.

Vectors

The resolved part of $\mathbf{a}$ in the direction of $\mathbf{b}$ is $\dfrac{\mathbf{a.b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$$\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b}.(\mathbf{c} \times \mathbf{a}) = \mathbf{c}.(\mathbf{a} \times \mathbf{b})$$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \; (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a.n}$

The plane through non-collinear points A, B and C has vector equation

$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector $\mathbf{a}$ and parallel to $\mathbf{b}$ and $\mathbf{c}$ has equation

$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$.

Hyperbolic functions

$\cosh^2 x - \sinh^2 x \equiv 1$

$\sinh 2x \equiv 2\sinh x \cosh x$

$\cosh 2x \equiv \cosh^2 x + \sinh^2 x$

$\operatorname{arcosh} x \equiv \ln\{x + \sqrt{x^2 - 1}\} \qquad (x \geqslant 1)$

$\operatorname{arsinh} x \equiv \ln\{x + \sqrt{x^2 + 1}\}$

$\operatorname{artanh} x \equiv \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Differentiation

f(*x*)	**f′(*x*)**
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\dfrac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

f(*x*)	$\int \mathbf{f}(x)\ \mathrm{d}x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\dfrac{x}{a}\right), \quad \ln\{x+\sqrt{x^2-a^2}\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\dfrac{x}{a}\right), \quad \ln\{x+\sqrt{x^2+a^2}\}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\operatorname{artanh}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Arc length

$$s = \int \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, \mathrm{d}x \qquad \text{(cartesian coordinates)}$$

$$s = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\, \mathrm{d}t \qquad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \, \mathrm{d}s = 2\pi \int y \sqrt{\left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\, \mathrm{d}x$$

$$= 2\pi \int y \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\, \mathrm{d}t$$